Physical Geology
Horton's Laws

Name_________________

PLAYFAIR'S LAW (1802)

Every river appears to consist of a main trunk, fed from a variety of branches, each running in a valley proportioned to its size, and all of them together forming a system of valleys connecting with one another, and having such a nice adjustment of their declivities* that none of them join the principal valley either on too high or too low a level; a circumstance which would be infinitely improbable if each of these valleys were not the work of the stream which flows in it.

*slopes or gradients

What is Playfair's point here?

HORTON'S LAW OF STREAM NUMBERS (1940's)

The numbers of streams of successively lower orders in a given basin tend to form a geometric progression, beginning with a single trunk segment of the highest order, and increasing according to a constant bifurcation ratio.*

*splitting, or branching

The ratio of a geometric progression is found by dividing one term of the progression by the next lower term. It is the number, therefore, by which each term is multiplied to get the next term.

The *bifurcation ratio* is found by dividing the number of segments in one order by the number of segments in the next higher order. For instance, if there are 81 1st order segments and 27 2nd order segments, the bifurcation ratio would be equal to _______ / _______, or _______.

HORTON'S LAW OF STREAM LENGTHS (1940's)

The cumulative mean lengths of stream segments of successively higher orders tend to form a geometric progression beginning with the cumulative mean length of the first order segments and increasing according to a constant length ratio.

Mean Length: The average length of the segments of a given stream order. This value is calculated by dividing the total length of segments of a given order by the number of segments of that order.

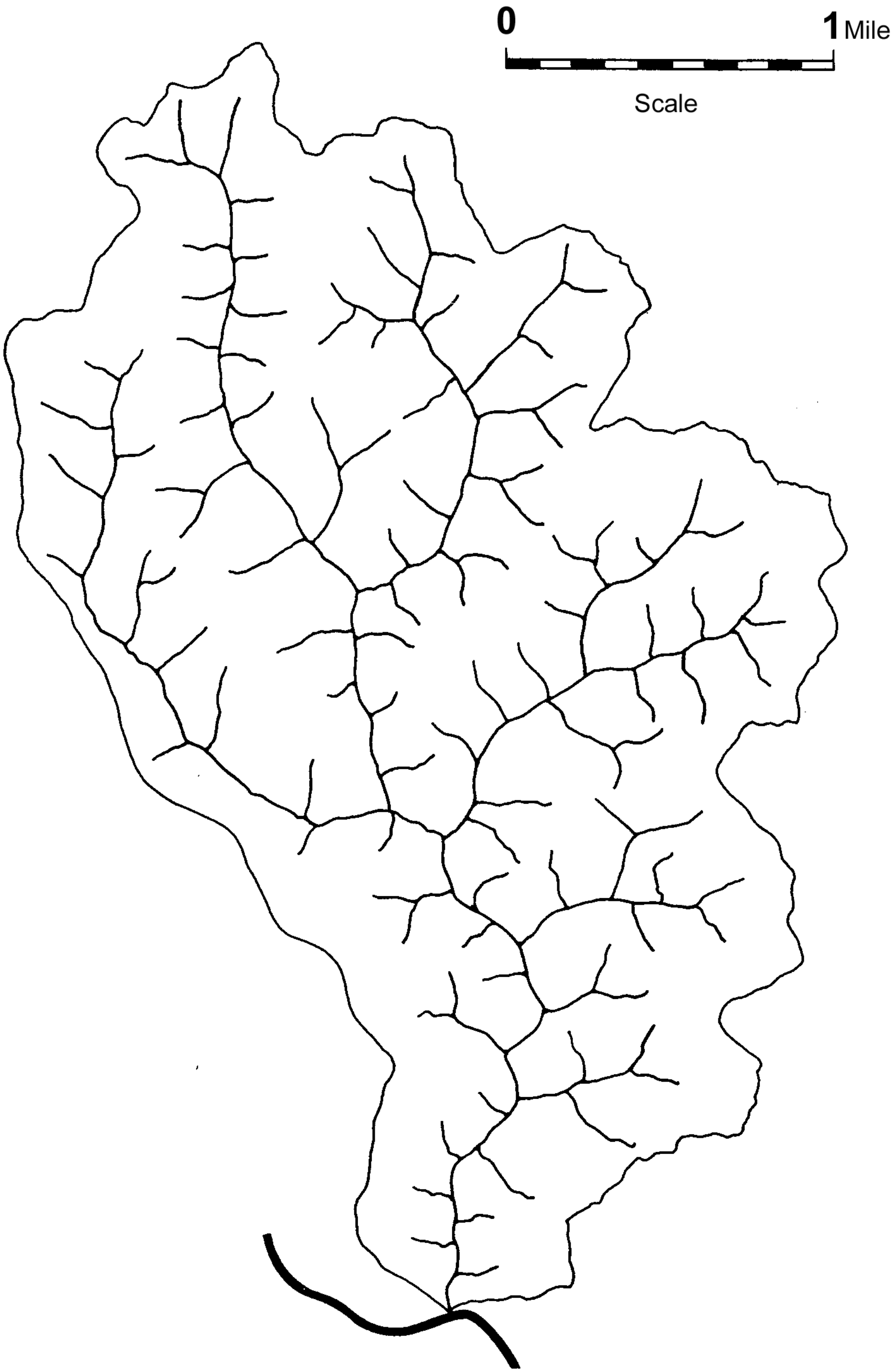
Cumulative Mean Length: The sum of the mean length of a given order and the mean lengths of all lower orders.

Length Ratio: The ratio of the cumulative mean length of the streams of one order to the cumulative mean length of the next lower order.

Order	Mean Length	Cumulative Mean Length	Length Ratio
1	0.2	_______	
			_______
2	0.7	_______	
			_______
3	1.9	_______	

HORTON'S OTHER LAWS

Horton found that other aspects of drainage systems revealed similar mathematical patterns. Among other things, he measured stream slopes, and the area of stream basins. In every case he described the observed patterns as geometric *pro*gressions (not *re*gressions), and calculated all the ratios of those progressions, therefore, as values greater than 1.


0
1 Mile
Scale

The map on the preceding page (page 2) represents a dendritic drainage pattern developed on the horizontal sedimentary strata of the Appalachian Plateau of Kentucky.

PART 1: LAWS OF STREAM NUMBERS AND LENGTHS

a. Assign orders to all segments of the stream system. Select several different colors and assign a color to each order. Find and mark in color all 1st order segments. Then find and mark all 2nd order segment, followed by the 3rd, and so forth. Prepare a key to your color code.

b. Count the number of segments of each order and record that data on Table 1 below. It makes sense to start at the first 1st order tributary on the west side of the "discharge end" of the basin, and work your way up and around the basin. Count carefully.

TABLE 1

Stream Order u	Number of Segments N_u	Bifurcation Ratio R_b
1	_______	
		_______
2	_______	
		_______
3	_______	
		_______
4	_______	

c. Plot the numbers of segments of each order on the graph on page 4. Fit the points with a straight line, and label the line.

d. Is the straight line a good fit with the points on your graph?_________

e. Re-read Horton's Law of Stream Numbers, and explain how your data supports that law.

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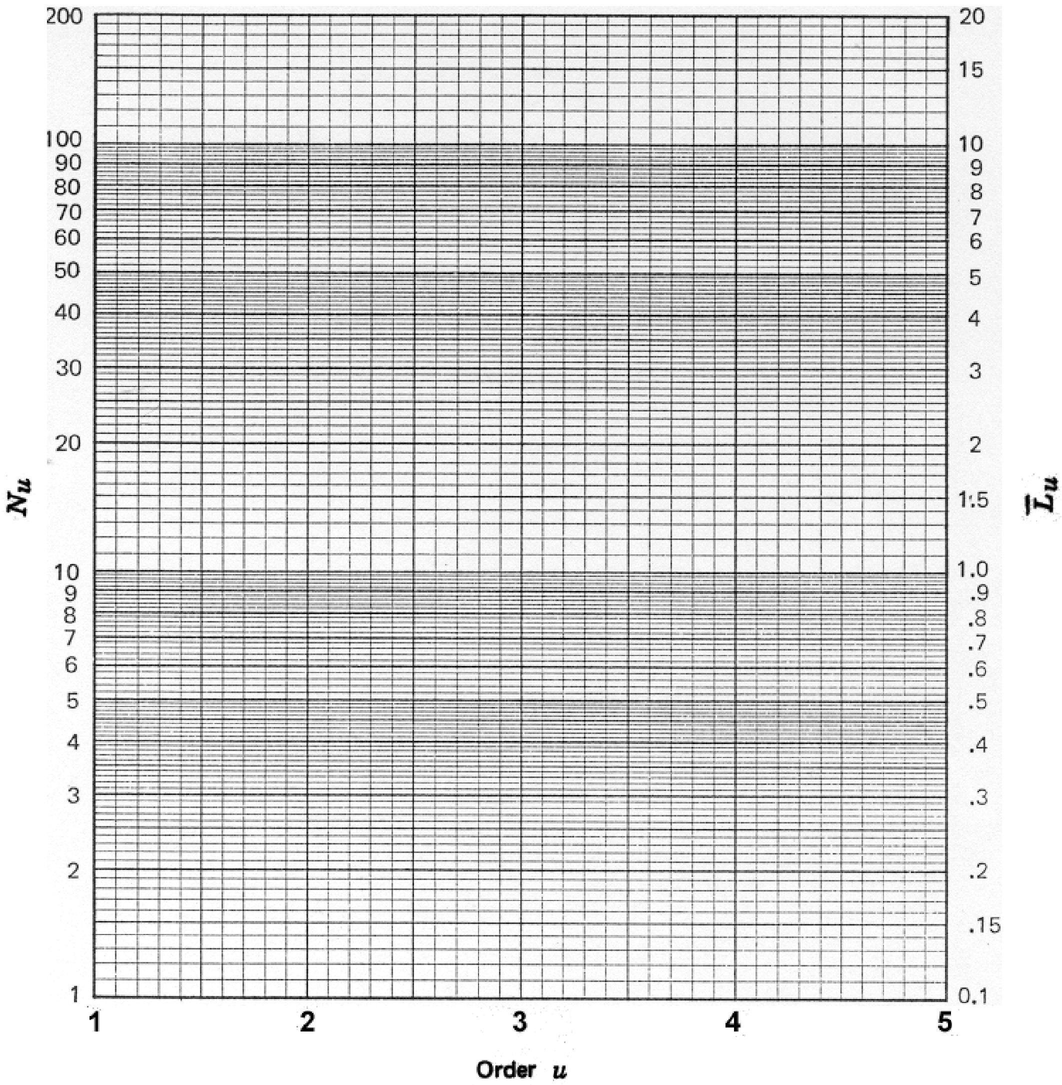
f. Devise a method to measure the total length of streams of the 1st order on the map. You may measure the lengths directly in miles, or in inches and convert to miles. Measure the total lengths of segments of each remaining order, and enter you data on Table 2 below.

TABLE 2

Stream Order U	Number of Segments N_U	Total Length of Streams of Order (in) L_U	Total Length (miles) L_U	Mean Length of Segments (miles)	Cumulative Mean Length (miles) $\overline{L}_U$	Length Ratio R_L
1	_______	_______	_______	_______	_______	
						_______
2	_______	_______	_______	_______	_______	
						_______
3	_______	_______	_______	_______	_______	
						_______
4	_______	_______	_______	_______	_______	


200
100
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80
70
60
50
40
30
20
N_u
10
9
8
7
6
5
4
3
2
1
20
15
10
9
8
7
6
5
4
3
2
1.5
$\bar{L}_u$
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.15
0.1
1
2
3
4
5
Order u

g. Divide the total length (miles) of each order by the number of stream segments of the order to obtain the mean length. Enter that data on Table 2 on the page 3.
h. Calculate and enter the cumulative mean length for each order. See page 1 for a discussion of cumulative mean length.
i. Calculate and enter the length ratios between each order.
j. Plot the cumulative mean length data on the graph on page 4 using the scale on the right hand Y-axis, and fit the plotted points with a straight line.
k. Explain how your data conforms with Horton's Law of Stream Lengths.

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PART 2: DRAINAGE DENSITY

a. Devise a way to measure the area of the basin mapped on page 2. Graph paper is available and useful for this purpose. You may measure directly in mi^2, or you may measure in inches or graph paper squares and convert to mi^2. Enter the data below:

Total Stream Length (mi)	Basin Area (mi^2)	Drainage Density (mi/mi^2)
______________	______________	________________

PART 3: HORTON'S LAWS – AN OVERVIEW

The tables below list morphometric data for Little Mill Creek, Ohio, a 4^{th} order basin with an area of 2.7 mi^2. The 4^{th} order basin is not complete, so values in parentheses should not be plotted.

TABLE 3

Stream Order U	Number of Segments N_U	Bifurcation Ratio R_b	Mean Length of Segments (mi)	Cumulative Mean Length (miles) $\overline{L}_U$	Length Ratio R_L
1	104		0.07	______	
		______			______
2	22		0.19	______	
		______			______
3	5		0.65	______	
		______			______
4	1		(1.2)	______	

a. Calculate and enter the bifurcation ratios, cumulative mean lengths, and the length ratios on Table 3 above.

b. Plot the "number of segments" data and the "cumulative mean lengths" on the proper graph on page 8 and fit them with a straight line.

c. Explain how the data in Table 3 supports Horton's Laws of Stream Numbers and Lengths.

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d. Recall that the data for the the 4th order is incomplete, and therefore the ratios calculated between the 3rd and 4th orders are suspect. Explain why this example would be inadequate to "prove" Horton's Laws.

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TABLE 4

Stream Order U	Mean Basin Area (mi^2) A_U	Area Ratio R_a	Mean Channel Slope (%) S_c	Slope Ratio R_s
1	0.025		0.37	
		_______		_______
2	0.12		0.12	
		_______		_______
3	0.58		0.04	
		_______		_______
4	(2.7)		(0.01)	

e. Using the data for Little Mill Creek in Table 4 above, calculate and enter the area ratios and slope ratios. Recall that the ratios are calculated for geometric progressions (not regressions) and are therefore greater than 1.

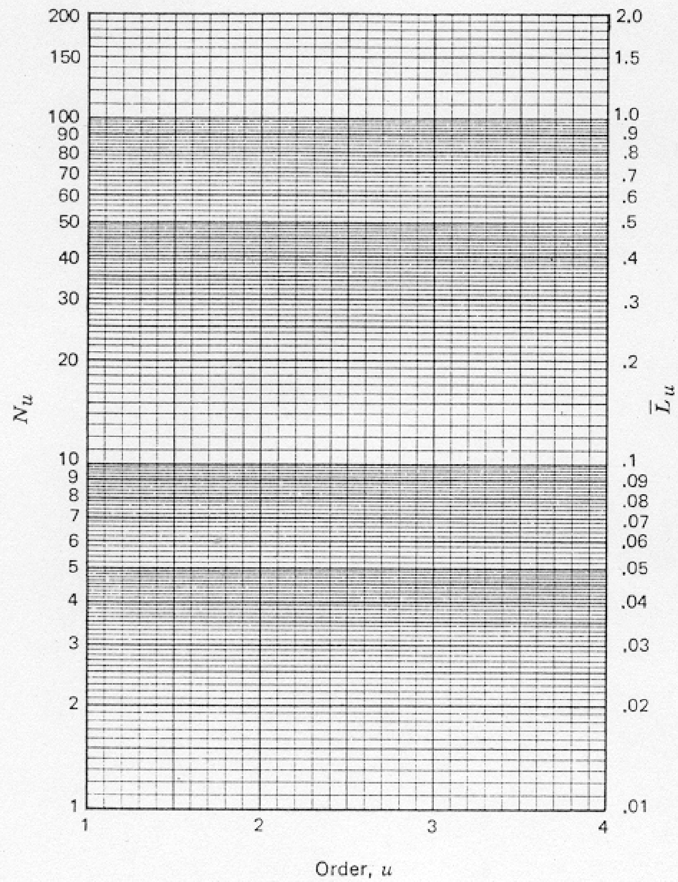
f. Plot the mean basin areas against stream order, and the mean channel slope against stream order on the proper graph on page 8. Fit the points with a straight line.

g. Assume that the patterns in the data in Table 4 are typical of many stream systems, and state the following in the style that Horton would have used:

The Law of Stream Basin Areas:

The Law of Stream Channel Slopes:

h. Plot the mean basin areas (Table 4) against the cumulative mean lengths (Table 3) for orders 1, 2 and 3 on the graph on page 8. Fit the points with a straight line. What, exactly, is the significance of that line?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6
5
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3
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N_u
2.0
1.5
1.0
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.03
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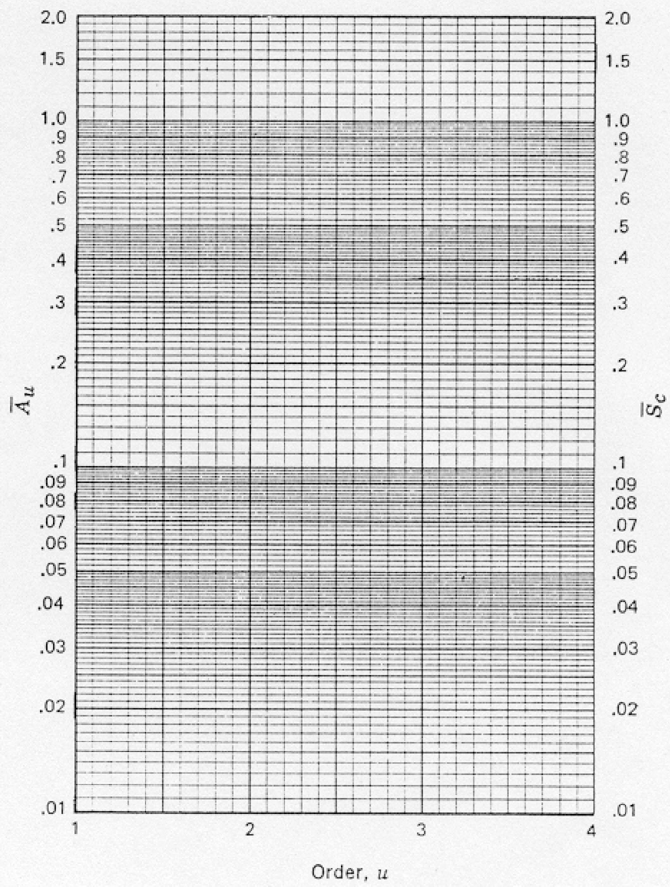
.01
$\overline{L}_u$
1
2
3
4
Order, u


$\overline{A}_u$
2.0
1.5
1.0
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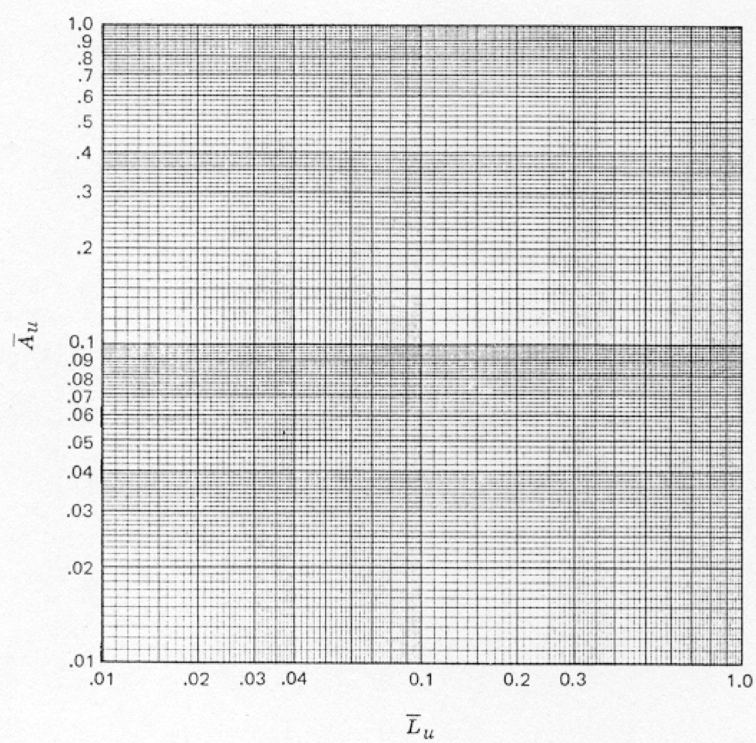
$\overline{S}_c$
1
2
3
4
Order, u


1.0
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0.3
1.0
$\overline{L}_u$